Fine grain
Topology and Set theory

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Definitions

Definition

C$_k \forall k \in \mathbb{Z}$, we assign $C_k(O_k, \rho_k)$, with $O_k(-2^k \rho_0, 0)$ and $\rho_k = 2^k \rho_0$.

Definition

B$_k \forall k \in \mathbb{Z}$, B_k is a form supported by C_k. The origin is O, and the extremity is $S_k \notin B_k$.

1. $\mathit{if}\,(k > 0)$ As we can make a recovery of the circle C_k many times as a loop.
2. $\mathit{if}\,(k \leq 0)$ we are drawing an arc of a circle.

Basic case

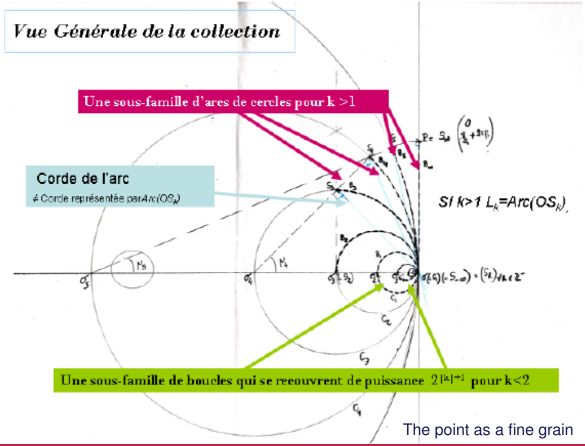
We want to determinate $(M_k) \in \mathcal{P}$ as $\mathbf{O_k M_k} = \cos(\theta_k)\mathbf{O_k T_k}$, where $\theta_k = \widehat{OO_k T_k}$.

Hence, $\mathbf{O_{k-1} M_k} = \rho_{k-1}(\cos 2\theta_k \mathbf{i} + \sin 2\theta_k \mathbf{j})$. Let $\theta'_k = 2\theta_k$. The relationship between the definition set of θ_k and θ'_k is $[0; 2\pi[\rightarrow [0; 4\pi[$.

So M_k will describe C_{k-1} sweeps two times.

$O_{2^{n_\infty}}$

if we are applying the basic case for all $(C_\ell)_{\ell \leq 1}$, we draw the point O sweeps 2^{n_∞}.



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Fine-grain topology

Thus, the previous result is showing us a problem with the axiomatic from Euclide :

" A point is that which has no part".

We will define another approach of the point with the fine grain topology.

Structure

- Scale.
- Locus.
- Fine-grain.
- NA (agraindissement level).
- Cardinal : $\#$.
- Measure of the lenght or width : μ.

Power of the continuum

$$\boxed{\#(\mathcal{C}) = 2^{n_\infty}}$$

Reny's Information

Variation of the size of the grain

Let's call N the number of symbol of a system Ω.

$$\boxed{\mathcal{I} = \log_2(N)}$$

We can apply to the variation of the size of the grain.

$$\boxed{g^{m+1} = 2^{n_\infty} g^m}$$

Fractal of the grain

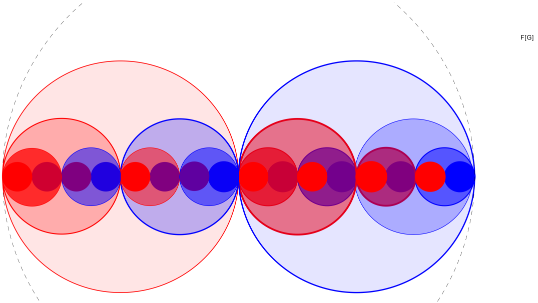


Figure: Fractal.

$$\text{dim} = -\frac{\ln(n_\infty)}{\ln 2}$$

Grain and Map's dilation

$$g_{di}^{m} = g^{m+1}$$

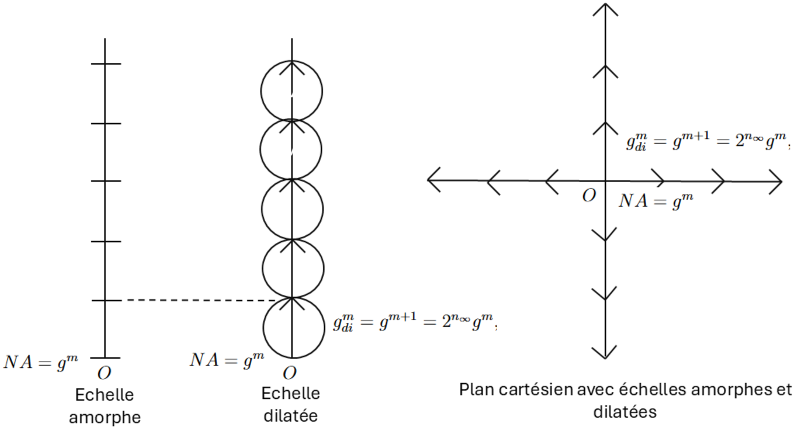
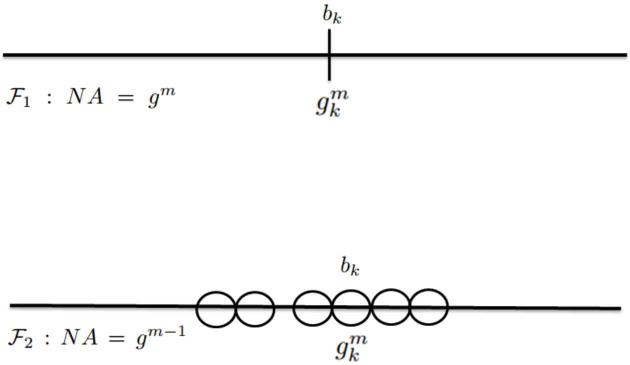


Figure: Dilations.

Enumerability of the real and Continuum hypothesis



Axiom of choice

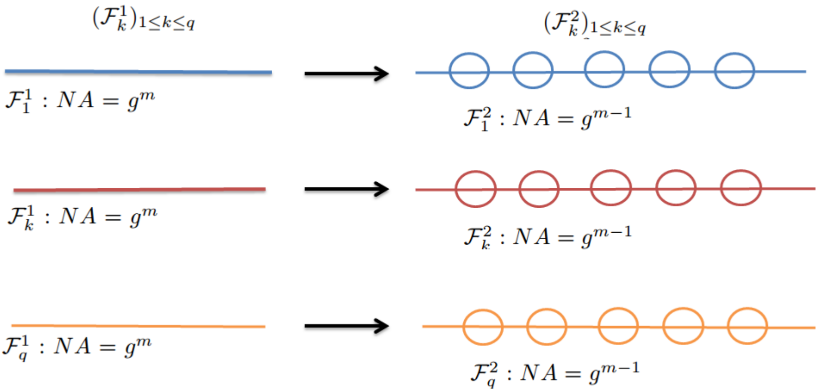


Figure: Axiom of choice.

Function of choice

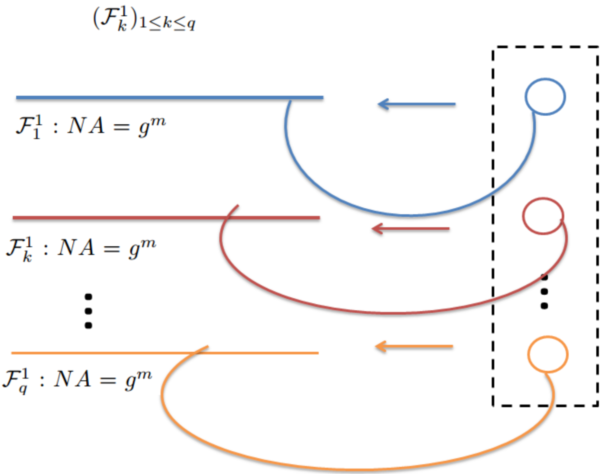


Figure: Function of choice.

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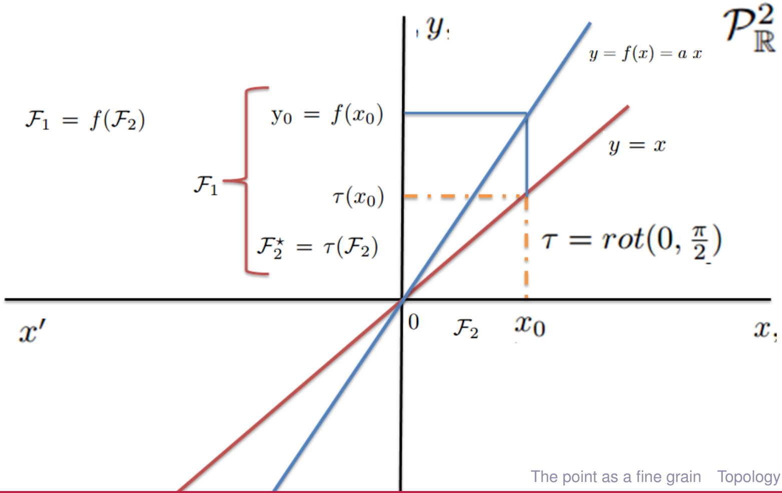
Paradox of reflexivity

Proper part is not taller than partial part and Dedekind theorem for infinite set is false



Encapsulation

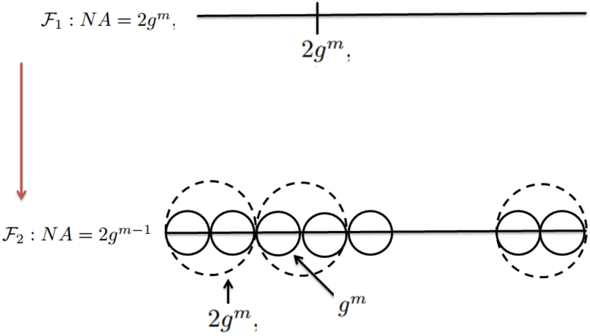


Figure: Encapsulation of the grain.

Application

The encapsulation of two radial grain is involving a grain with concentric dilation. That's allow us to assign a kind of grain which is equal to a transverse section of the fibration of S^3.

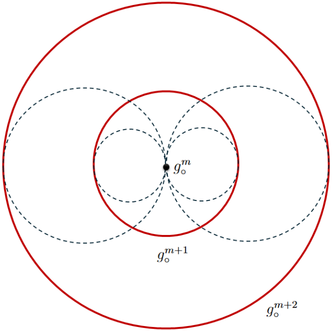


Figure: Concentric dilation.

Complex grain

As we work in $\mathcal{P}_{\mathbb{C}}^{\star}$, we are leading to identify $\frac{1}{2}g_{\circ}^{m+1}$ and $e^{\imath\theta}$ for $\theta \in [0; 2\pi[$.

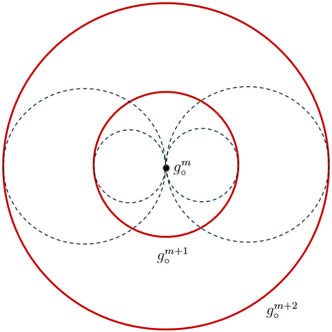


Figure: Concentric dilation.

Application

The infinite circle and design of a square grain $g_{\square}^{m+2} = 4g_{\circ}^{m+2}$

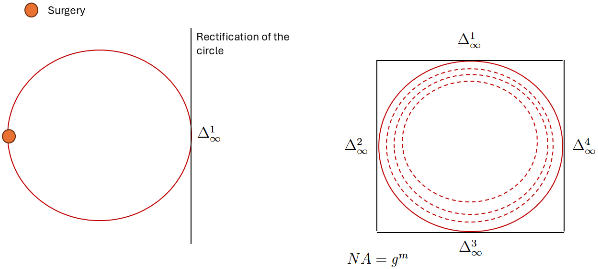


Figure: Square grain.

With $NA = g^m$, $4g_{\circ}^{m+2} \simeq g_{\square}^{m+2}$, $4g_{\circ}^{m+1} \neq g_{\square}^{m+1}$, $4g_{\circ}^{m} = g_{\square}^{m}$.

Dedekind's cut

An impossible construction

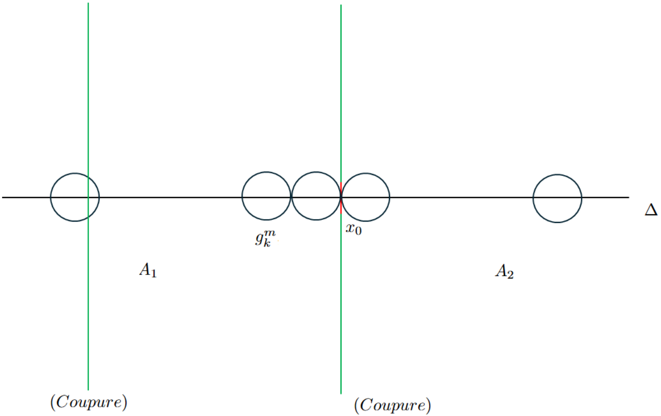


Figure: Dedekind's cut.

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Generalization of Dedekind's cut

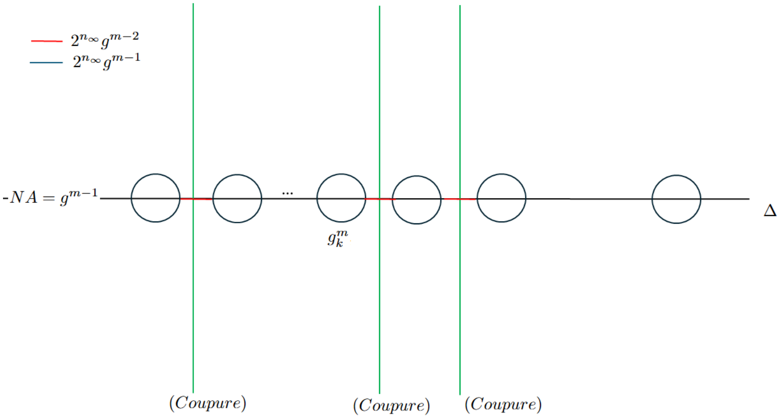

Figure: Dedekind's cut.

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