Information Theory in the Grain Program

Hartley, neguentropy, stratal entropy, spectral analysis, coin toss and perception

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Overview

Definition. For a set of N distinguishable symbols,

$$\mathcal{I} = \log_2 N \qquad (2^{\mathcal{I}} = N).$$

- Measures a *capacity of distinction* (in bits).
- Morphism: $\mathcal{I}(K_1 \times K_2) = \log_2 K_1 + \log_2 K_2 = \mathcal{I}(K_1) + \mathcal{I}(K_2)$.
- Continuous extension: $dI = \frac{1}{\ln 2} \frac{dK}{K}, \quad d^2 I = -\frac{1}{\ln 2} \frac{dK}{K^2}$.

Units

Count in bits ($\log_2$); *differentiate* in nats ($\ln$), where $dI = dK/K$ has no $1/\ln 2$ factor. Same order, constant $\ln 2$ apart.

The stratal hierarchy, read by the logarithm

Grains at level j: $N = 2^{jn_\infty}$ symbols. The logarithm sends cardinals to *exponents*, where comparison is licit:

$$\mathcal{I}(C_{kn_\infty}) = kn_\infty \; < \; \mathcal{I}(C_{jn_\infty}) = jn_\infty \; < \; \mathcal{I}(\Delta_\infty) = \aleph_0 = \sup \mathcal{I}.$$

- First two: *hyperfinite* integers (stratal levels).
- Last: the cardinal $\aleph_0$, reached *by promotion* — not by finer counting.
- Without the log, one would compare 2^{kn_∞} (hyperfinite) to $2^{\aleph_0}$ (continuum): incomparable. The log resolves it.

From Hartley to Shannon by composition

Message of N symbols. *Unstructured* (NS): no sub-segments. *Structured* (S): partition into $N_1, \ldots, N_n$, $\Pr_i = N_i/N$.

Theorem

$$\Delta\mathbb{E} = \mathbb{E}_S - \mathbb{E}_{NS} = \sum_i \Pr_i \log_2 N_i - \log_2 N = \sum_i \Pr_i \log_2 \Pr_i = -\mathcal{H}_{\mathrm{Shannon}}.$$

Structuring lowers the expected Hartley information by $\mathcal{H}$. This deficit is the raw material; the *true* neguentropy is its distance to a maximum (next slide) — Shannon bits, *not* thermodynamic k_B.

Two neguentropies, both relative (deficits to a maximum)

A neguentropy is a *deficit to a maximum* — a Kullback–Leibler divergence. Both read the same way:

Bias (departure from uniform):

$$\mathcal{N}_{\text{bias}} = \mathcal{H}_{\max} - \mathcal{H}_{\text{real}} = D_{\mathrm{KL}}(p \,\|\, u_m).$$

Zero iff equiprobable symbols.

Composition (imbalance of the partition):

$$\mathcal{N}_{\text{compo}} = \log_2 n - \mathcal{H}(\mathrm{Pr}) = D_{\mathrm{KL}}(\mathrm{Pr} \,\|\, u_n).$$

Zero iff balanced partition.

Relative neguentropy

$\mathcal{H}(\mathrm{Pr})$ alone is an *entropy*, not a neguentropy. The neguentropy of composition is the *deficit* $\log_2 n - \mathcal{H}$ to the balanced partition — exactly the same form as the bias. (Correlation between positions is a third, distinct quantity: total correlation $\mathcal{N}_{\mathrm{corr}}$.)

Correlation, and measurement on DNA

A *third* quantity, on the level axis: correlation between positions,

$$\mathcal{N}_{\mathrm{corr}} = \sum_{\ell} \mathcal{H}(X^{\ell}) - \mathcal{H}(X^{1}, \ldots, X^{L}) \;\geq 0, \qquad \text{zero iff independent.}$$

Measured on DNA

Level	$\mathcal{H}_{\mathrm{real}}$	Neguentropy
bases (human), bias	1.977	0.023 /base
bases (*Plasmodium*), bias	1.722	0.278 /base
codons, correlation	5.774	0.156 /codon

Bias and correlation are distinct sources of structure — do not add them naively.

Convexity of the relative neguentropy (in $\Pr$)

With the relative definition $\mathcal{N}_{\text{compo}} = \log_2 n - \mathcal{H}(\Pr)$, the sign settles cleanly:

$\mathcal{H}$ concave $\Rightarrow$ $\mathcal{N}$ convex

$\phi(t) = -t \log_2 t$ has $\phi'' < 0$: $\mathcal{H}$ strictly concave. A constant minus a concave is **strictly convex**:

$$\mathcal{N}_{\text{compo}} = \log_2 n - \mathcal{H} \quad \text{convex in } \Pr.$$

Meaning: minimum of order

Convex $\Rightarrow$ minimum inside, at the balanced partition ($\mathcal{N} = 0$), growing toward concentrated ones. Order is minimal at balance, maximal at concentration — as a neguentropy should behave.

Convexity is in $\Pr$ (the distribution of sizes), not in the composition level k.

The two axes: a saddle

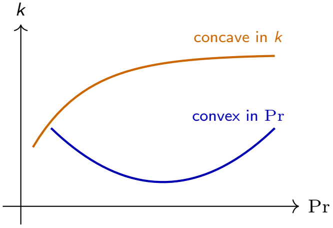
The neguentropy of composition has *two* variables — with opposite curvatures. This dissolves the "convex, so no saturation" paradox.

Axis Pr (fixed level)

$\mathcal{N} = \log_2 n - \mathcal{H}(\mathrm{Pr})$ is **convex**: minimum at balance, growing toward concentrated partitions.

Axis k (level, correlations)

$\mathcal{N}_{\mathrm{corr}}(k) = \sum_{j \le k} I_j$ is **increasing, concave, saturating** — *iff* long-range correlations weaken ($I_j \downarrow$).



$f(x, y) = x^2 - y^2$: convex one way, concave the other.

"Convex in Pr" does not imply "non-saturating in k": different axes.

Curvature in k is set by the increments

$$\Delta^2\mathcal{N}_{\mathrm{corr}}(k) = I_{k+1} - I_k.$$

- correlations weaken ($I_{k+1} \le I_k$): **concave**, saturating to $\sum_j I_j$ if summable;
- pure Markov-1 (I_j constant): **linear**, no saturation.

k	1	2	4	6	8	$\to \infty$
$\mathcal{N}_{\mathrm{corr}}(k)$	0.083	0.129	0.168	0.179	0.183	0.184
$\Delta^2\mathcal{N}_{\mathrm{corr}}$	—	-0.037	-0.011	-0.003	-0.001	$\to 0$

Saturation is a property of the *source* (finite memory), not a law of composition.

Composition / decomposition asymmetry

Partition entropy $\mathcal{H}(p) = -\sum_i p_i \log_2 p_i$ (labeling information).

Same cost, asymmetric reversibility

$$|\text{compose}| = |\text{decompose}| = \mathcal{H}(p),$$

but **compose** (acquire labels) is *reversible*; **decompose** (erase labels) is *irreversible* (Landauer).

Non-free cycle: $S \to NS \to S$ dissipates $\mathcal{H}(p)$ bits (e.g. 1.648 bit/symbol) — signature of irreversibility.

Link to the grain program

Promotion $\mathrm{st} : \widetilde{\mathbb{R}}_{\mathrm{fin}} \to \mathbb{R}$ erases infinitesimals: it is a *decomposition* — irreversible and dissipative. Refining into grains is composition (reversible).

Stratal entropy of the continuum

A single real point needs infinite information: $\mathcal{I}_{\text{abs}} = +\infty$. At resolution 2^{-m}:

$$\boxed{\mathcal{H}_{\text{strate}}(m) = \underbrace{m}_{\text{resolution}} + \underbrace{h}_{\text{shape}}}$$

- m (resolution) *diverges* — the fineness *erased* by promotion; analogue of grain finesse jn_∞.
- h (differential entropy) is *finite*, may be negative — informational analogue of the *standard part*.

Density as cause of divergence

Density (ordinal: no successor) $\Rightarrow$ unbounded refinement $\Rightarrow$ $m \to \infty \Rightarrow \mathcal{I}_{\text{abs}} = +\infty$. Density *explains* the infinite cardinal.

Support vs structure

Support information

$$\mathcal{I}_{\text{supp}} = \log_2 |E| = \aleph_0$$

Same for every 1D continuous form — *blind* to shape.
$[0,1]$ and the Cantor set: same $\mathcal{I}_{\text{supp}}$.

Structure information
Cost to specify *which* form. Finite iff Fourier coefficients decay:

$$|c_n| \sim n^{-\alpha}, \ \alpha > 1 \Rightarrow \mathcal{I}_{\text{struct}} < \infty.$$

Form	decay	$\mathcal{I}_{\text{struct}}$
analytic (cos)	exponential	~ 9 bits
triangle (kink)	n^{-2}	~ 20 bits
square wave (jump)	n^{-1}	$\to \infty$

Total information: cardinal non-additivity

Chain rule: $\mathcal{I}_{\text{total}} = \mathcal{I}_{\text{supp}} + \mathcal{I}(\text{struct} \mid \text{supp})$.

The cardinal absorbs the addition

$$\aleph_0 + \text{finite} = \aleph_0, \qquad \aleph_0 + \aleph_0 = \aleph_0 \;\Rightarrow\; \mathcal{I}_{\text{total}} = \aleph_0 = \mathcal{I}_{\text{supp}}.$$

A wild form "contains more" *qualitatively*, but the cardinal plateaus at $\aleph_0$. Strict comparison is restored **only at finite fineness**:

$$\mathcal{I}^{\text{wild}}_{\text{strate}}(m) > \mathcal{I}^{\text{smooth}}_{\text{strate}}(m) \quad (214 > 9 \text{ bits}).$$

The grains are what make visible what the cardinal erases.

A paradox of information and set theory

Same cardinal, same Hartley content

Take two continua of dimension 1:

- $\mathfrak{c}$ — the *bare* line;
- $\mathfrak{c}'$ — a *structured* continuum (variation, convexity).

Both have the cardinal of the continuum, so

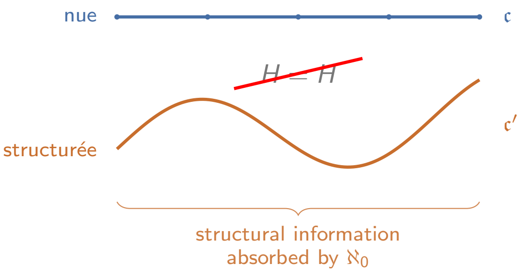
$$H(\mathfrak{c}) = H(\mathfrak{c}') = \log_2(2^{\aleph_0}) = \aleph_0.$$

Yet $\mathfrak{c}'$ visibly carries more

Its shape, its variation, its convexity — an entire layer of *structural* information. Where did it go?

The question

How can two objects of *unequal* structure carry the *same* information? Is the point itself a well-defined carrier of information at all?



The cardinal is *saturated*: $\aleph_0 + \kappa = \aleph_0$. It cannot see the difference.

The continuum paradox: absorption is idempotence, not smallness

Same cardinal, unequal structure

The bare line $\mathfrak{c}$ and a structured continuum $\mathfrak{c}'$ share

$$\mathcal{I}(\mathfrak{c}) = \mathcal{I}(\mathfrak{c}') = \log_2(2^{\aleph_0}) = \aleph_0,$$

yet $\mathfrak{c}'$ visibly carries more — its structure κ.

Structure *saturates* the bound

Universal bound: any relation on $\mathfrak{c}'$ has $\kappa \le \log_2(2^{\aleph_0})^2 = \aleph_0$.
Attained (regular $\mathfrak{c}'$): the Fourier signal segments into $N' = \aleph_0$ symbols, so $\kappa = \log_2(\aleph_0) = \aleph_0$.

The point of the paradox

Structure and support are *both* $\aleph_0$ — and still

$$\underbrace{\aleph_0}_{\text{support}} + \underbrace{\aleph_0}_{\text{structure}} = \aleph_0.$$

Absorption is **idempotence**, not a smallness of κ: a *maximal* structure is swallowed. The fault is in the instrument (the cardinal), not in the object.

Cardinal caution

$2^{\aleph_0} \cdot 2^{\aleph_0} = 2^{\aleph_0+\aleph_0} = 2^{\aleph_0}$ — no jump, since $2\aleph_0 = \aleph_0$.

The grain replaces the saturated cardinal by non-absorbing stratal depth (bits).

Non-affinability of the point: information as a certificate

The cardinal paradox

The bare line $\mathfrak{c}$ and a structured continuum $\mathfrak{c}'$ share the same Hartley content

$$H(\mathfrak{c}) = H(\mathfrak{c}') = \log_2(2^{\aleph_0}) = \aleph_0,$$

since $\aleph_0$ *absorbs* structure: $\aleph_0 + \kappa = \aleph_0$.

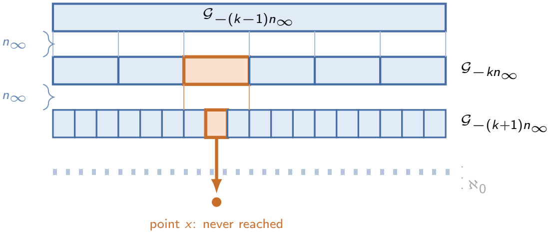
The grain lifts the absorption

One refinement costs $\Delta H = \log_2(2^{n_\infty}) = n_\infty$ bits (non-absorbing), hence

$$\boxed{I(\mathfrak{c}') = I_{\text{support}} + I_{\text{struct}|\text{support}}} \;\Rightarrow\; I(\mathfrak{c}') > I(\mathfrak{c}).$$

The point

No sub-structure to refine: $H(\{x\}) = 0$, $H(\text{limit}) = \aleph_0$, nothing between. *Non-affinability* $\Rightarrow$ *ideality.*



Each refinement ρ splits a grain into 2^{n_∞} children (n_∞ bits/step); the point is the limit no stratum closes.

Informational deformation under a map

On a *continuous* informational domain, a map g deforms the local measure (grain size) by its Jacobian $|g'|$.

The Jacobian, read in bits

A grain of size ε at x maps to size $|g'(x)|\varepsilon$ at $y = g(x)$:

$$\Delta\mathcal{I}_g(x) = \log_2 \frac{\varepsilon'}{\varepsilon} = \log_2 |g'(x)| \quad \text{bits.}$$

The *multiplicative* Jacobian becomes an *additive* deformation.

Reciprocity: a conservation law

$$\Delta\mathcal{I}_{g^{-1}}(y) = -\Delta\mathcal{I}_g(x), \qquad y = g(x)$$

What a map gains in local information, its inverse gives back — bit for bit. The round trip deforms nothing.

Chain rule = additivity

$$\Delta\mathcal{I}_{g_2 \circ g_1} = \Delta\mathcal{I}_{g_2} \circ g_1 + \Delta\mathcal{I}_{g_1}$$

A morphism: composition $\to$ addition of bits (like Hartley on products).

$\Delta\mathcal{I}_g$ is differential entropy (signed); it lives in the "form" layer that promotion keeps.

Spectral reading of the deformation

A physical signal deformed by g: the amplitude is weighted by the Jacobian. Take the logarithm — the deformation becomes *additive*.

Log separates, Fourier isolates

$$\log_2 s_{\text{def}}(y) = \underbrace{\log_2 s\left(g^{-1}(y)\right)}_{\text{reparametrised signal}} + \underbrace{\Delta\mathcal{I}_{g^{-1}}(y)}_{\text{deformation}}$$

$\mathcal{F}$ is linear: if the two spectra occupy *disjoint bands*, filtering recovers $\Delta\mathcal{I}_{g^{-1}}$, hence $-\Delta\mathcal{I}_g$ (conservation).

Separability is *the* condition

Regime	RMS recovery error
Separable (freq 3 vs 60)	0.0%
Overlapping (58 vs 60)	100%

Two conditions

(i) spectral separability (mandatory);
(ii) small deformation, for the direct spectral match (g^{-1} reparametrises).

The cepstrum: neguentropy of composition, per scale

"Log then Fourier to separate a product" is exactly the *cepstrum* — $\Delta\mathcal{I}_g = \log_2|g'|$ lives in the cepstral domain.

Integrated deformation = neguentropy of composition

$$h(Y) = h(X) + \mathbb{E}\left[\log_2|g'|\right], \qquad \mathcal{N}_g = h(Y) - h(X) = \mathbb{E}[\Delta\mathcal{I}_g].$$

The deformation, integrated over the domain, *is* the neguentropy injected by g (a change of stratum: source $\to$ deformed).

Per quefrency

By Parseval, each frequency band contributes its share:

$$\mathcal{N}_g = \sum_B \mathcal{N}_g^{(B)}.$$

The cepstrum *ventilates* the neguentropy scale by scale.

Worked value

$X \sim \mathcal{U}[0,1]$, $g(x) = x^2$:

$$\mathcal{N}_g = \int_0^1 \log_2(2x)\,dx = -0.44 \text{ bit}.$$

Negative: x^2 *contracts* near 0.

Bridge: deformation (Jacobian) $\leftrightarrow$ cepstrum $\leftrightarrow$ neguentropy of composition — three readings of one object.

Weierstrass: stratifiable though nowhere differentiable

$$W(x) = \sum_{n=0}^{\infty} a^n \cos(b^n \pi x), \quad 0 < a < 1, \ ab > 1.$$

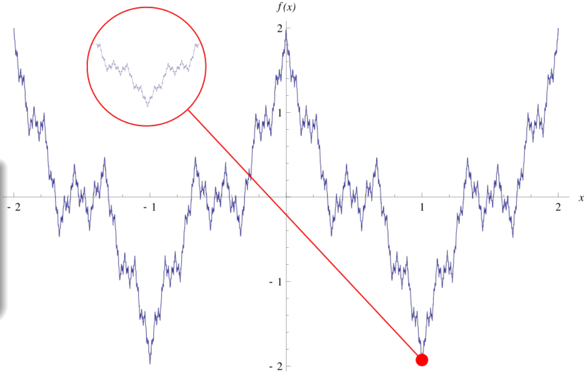
Continuous everywhere, differentiable nowhere. Its spectrum is *full* ($a^n > 0$ for all n), so it **saturates**: $\kappa = \aleph_0$.

Two sums decide — oppositely

$$\sum_n a^n = \frac{1}{1-a} < \infty \quad (\ell^1 : \ \text{stratifiable})$$

$$\sum_n (ab)^n = \infty \quad (ab > 1 : \ \text{nowhere diff.})$$

N	$\sum a^n$	$\sum (ab)^n$
2	1.750	4.75
6	1.984	32.2
10	1.999	171



Stratal robustness

The grain frame requires only ℓ^1 amplitudes (segmentation), *not* differentiability. It reconstructs W where classical calculus fails: the pathological function becomes a *regular* stratal object.

Two regimes — and a bridge to transcendence

The dividing line: affinability

Information lives in *stratal depth*, not in isolated positions. An object carries information to the extent that it can be *refined*.

	Continuum	**Point**
Sub-structure	yes	none
Refinement ρ	non-trivial	degenerate
Information	$I_{\text{support}} + I_{\text{struct}\mid\text{support}}$	0 or $\aleph_0$
Status	physical	ideality

Structure converts into stratal depth; the point resists conversion.

Conjecture (Bridge to Liouville)

A *finite* stratal address $\Leftrightarrow$ **algebraic** (a dyadic reached in finitely many steps).

An *irreducibly infinite* address $\Leftrightarrow$ **transcendental**: π, $\ln 2$ are never grain positions.

finite refinement (algebraic) $\xrightarrow{\text{deeper}}$ infinite refinement (transcendental) $\xrightarrow{\text{limit}}$ never closed (ideal point)

The question

A tempting idea

A coin toss produces sequences of heads and tails — runs, patterns. Isn't this the very example of "structure appearing", i.e. a *neguentropy of composition*?

We will see the answer is **no** — and that this "no" makes the coin the reference point against which all neguentropy is measured.

The balanced coin has zero neguentropy

Proposition

For a fair coin ($p = \frac{1}{2}$):

$$\mathcal{H} = -\tfrac{1}{2}\log_2 \tfrac{1}{2} - \tfrac{1}{2}\log_2 \tfrac{1}{2} = 1\ \text{bit} = \mathcal{H}_{\max}, \qquad \mathcal{N} = \mathcal{H}_{\max} - \mathcal{H} = 0.$$

- The fair coin is the case of *maximal* entropy.
- One cannot be more disordered than equiprobable.
- So the coin is the **etalon** — the "zero of structure" — not an example of neguentropy.
- A fair toss *learns* exactly 1 bit: the maximum, not an excess of structure.

The bionomial distribution

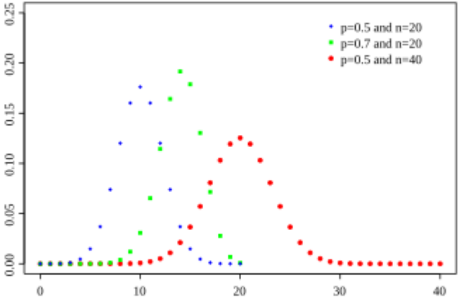


Figure: Probability mass function

Law

$f(k, n, p) = \Pr(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$

Bias creates measurable neguentropy

As soon as $p \neq \frac{1}{2}$, the bias *is* structure, measured in bits:

$$\mathcal{N} = \mathcal{H}_{\mathrm{max}} - \mathcal{H}(p).$$

p (heads)	0.6	0.8	0.99
$\mathcal{H}$ (bits)	0.971	0.722	0.081
$\mathcal{N}$ (bits)	0.029	0.278	0.919

The more loaded the coin, the larger the neguentropy — up to 1 bit for a deterministic coin.

Correlation: neguentropy without bias

Subtler: even with *no* bias ($p = \frac{1}{2}$), a *correlated* coin carries neguentropy. “Sticky” Markov coin, $P(\mathrm{H} \mid \mathrm{H}) = 0.9$, stationary $p = \frac{1}{2}$:

Proposition

$$\mathcal{H}_{\mathrm{marg}} = 1, \quad \mathcal{H}_{\mathrm{cond}} = 0.469, \quad \mathcal{N}_{\mathrm{compo}} = \mathcal{H}_{\mathrm{marg}} - \mathcal{H}_{\mathrm{cond}} = 0.531 \text{ bit/toss.}$$

- The marginal stays maximal ($p = \frac{1}{2}$, no bias).
- But *correlation* between successive tosses creates neguentropy: knowing the past reduces surprise.
- **Bias and memory are two distinct sources of structure.**

Support and structure of a sequence

- **Support** of n tosses: the 2^n possible sequences, $\mathcal{I}_{\text{supp}} = \log_2 2^n = n$ bits.
- **Structure** of one sequence: its compressibility, the Kolmogorov complexity $K(s)$ (shortest program producing s).

Sequence ($n = 16$)	short description	K (order)
PPPP. . . P	"P×16"	small
PFPF. . . PF	"(PF) × 8"	small
random	none shorter than itself	$\sim n$

Random = incompressible

Proposition

A sequence is *Kolmogorov-random* iff it is *incompressible*: $K(s) \approx n$. At most 2^{n-c} sequences admit a description $\leq n - c$ bits — a fraction 2^{-c}.

compressible by $\geq c$ bits	$c = 1$	$c = 5$	$c = 10$
fraction of sequences (at most)	50%	3.1%	0.1%

Almost all sequences are incompressible. Randomness is not the *absence* of structure but *irreducible* structure: the whole support is needed to describe it ($\mathcal{I}_{\text{struct}} = \mathcal{I}_{\text{supp}} = n$).

The link between Act 1 and Act 3

Coherence

The fair coin *generates* Kolmogorov-incompressible sequences. The maximal chance of Act 1 *is* the incompressibility of Act 3.

- Act 1: fair coin = maximal entropy, $\mathcal{N} = 0$.
- Act 3: its typical output = incompressible, $K \approx n$.
- Two faces of the same fact: no structure to extract, nothing to compress.

Three roles of one object

Role	Quantity	Value
etalon (fair coin)	neguentropy	$\mathcal{N} = 0$ (maximal entropy)
bias / memory	neguentropy	$\mathcal{N} > 0$ (bias or correlation)
Kolmogorov	compressibility	random $\Leftrightarrow$ incompressible

Method

Every claim carries a computable *number*. The fair coin fixes the "zero of structure"; bias and correlation depart from it; Kolmogorov identifies randomness with incompressibility.

Nature's apply : neguentropy detects, it does not explain

What the logic gives

DNA has $\mathcal{N} > 0$, so it is **not** a random draw (proven fact). This structure *calls for* an explanation — something has biased and correlated the sequences.

What it does not give

Information theory states only *that* an explanation exists (the source is structured), not *which one*. The cause — selection, chemical constraints, phylogeny — belongs to biology, not to entropy.

Two guardrails

Neguentropy is a **detector of structure**, not an **explainer of cause**. And *non-random* $\neq$ *intentional*: it means only *not equiprobable* / *not independent*.

The acoustic wave

- **Support** = time interval $[0, T]$: $\mathcal{I}_{\text{supp}} = \aleph_0$, blind to the sound.
- **Structure** = spectrum: information classes sounds by *compressibility*.
- **Fineness** = hearing threshold (codec resolution).

Sound	components	$\mathcal{I}_{\text{struct}}$
pure tone (440 Hz)	1	7 bits
vowel "a"	3	17 bits
violin	29	90 bits
white noise	~ 4400	$\sim 20{,}700$ bits

Structure information = what an audio codec (MP3) keeps. *Exact*, not analogy.

Fechner–Weber as grain counting

Weber: smallest perceptible step $\Delta I / I = k$ (relative fineness). Coding ratios $\Rightarrow$ logarithm — same origin as Hartley.

Two stages

- **Sensation** $S = N(I) \propto \log(I/I_0)$ — grain count (Fechner's law).
- **Information of sensation** $\mathcal{I} = \log_2 N$ — bits to designate a level.

Audition: ~ 120 grains over the audible range, $\log_2 120 \approx 6.9$ bits — physiological order ($\sim$ 8–9 bits).

Guardrails: log-response modalities only (Stevens' power law elsewhere); the stimulus *carries* information relative to a repertoire, it is not information itself.

The informational oscillator

Angular recurrence $\dot{\theta}_k = 2\dot{\theta}_{k-1}$, $\dot{\theta}_1 = C$, gives dephasing between levels $\Delta\theta(t) = 2^{k-2}Ct$. With mutual information (bounded proxy) $I = \cos^2(\omega t)$:

Harmonic oscillator

$$\frac{d^2 I}{dt^2} = -(2\omega)^2\left(I - \tfrac{1}{2}\right), \qquad \text{stiffness } (2\omega)^2 = 2^{2k-2}C^2.$$

A *restoring force* toward $I_{\text{eq}} = \frac{1}{2}$; stiffness $\times 4$ per level. An **informational** oscillator (form analogy), *not* a physical wave. C is a free parameter — no external c injected.

What holds

- Hartley $\mathcal{I} = \log_2 N$; stratal hierarchy readable via the log.
- Composition theorem $\Delta\mathbb{E} = -\mathcal{H}$; *relative* neguentropy $\mathcal{N}_{\text{compo}} = \log_2 n - \mathcal{H} \geq 0$ (a true deficit, convex in Pr).
- **Two axes**: convex in Pr (imbalance), concave/saturating in k (correlations, if they weaken) — a saddle.
- Composition/decomposition: equal cost, asymmetric reversibility (Landauer); promotion = irreversible decomposition.
- Continuum: $\mathcal{H}_{\text{strate}} = m + h$; density causes divergence.
- Form: support ($\aleph_0$, blind) vs structure (regularity); grains restore comparison.
- Incarnations: audio compressibility; Fechner as grain counting.
- Dynamics: informational harmonic oscillator.

Method

Every claim carries a computable *number*. Where a charged word lacks a measurable quantity, it stays a metaphor.

Thank you

Information Theory in the Grain Program

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