Logique

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Conjugate of privation

$$
\begin{aligned}
\overline{(A \Leftrightarrow B) \setminus (A \Rightarrow B)} &= \Big((\overline{A} \overline{\Rightarrow} \overline{B}) \overline{\wedge} (\overline{A} \overline{\Leftarrow} \overline{B}) \Big) \overline{\setminus} (\overline{A} \overline{\Rightarrow} \overline{B}) \\
&= \Big((\overline{A} \Leftarrow \overline{B}) \vee (\overline{A} \Rightarrow \overline{B}) \Big) \overline{\setminus} (\overline{A} \Leftarrow \overline{B}). \\
\overline{(A \Leftrightarrow B) \setminus (A \Rightarrow B)} &= \overline{A \Leftarrow B} \\
&= \overline{A} \Rightarrow \overline{B}.
\end{aligned} \tag{1}
$$

$$\overline{\setminus} = \setminus$$

Conjugate of imply symbol

Relation of equivalence

Let $\mathcal{R} = \Leftrightarrow$ with the universe $\Omega_{\mathcal{R}} = \{0, 1\}$ such as $\overline{0}\mathcal{R}\ 1$ et $\overline{1}\mathcal{R}0$, où $\bar{x}$ designates the conjugate of x.

1. $0\mathcal{R}0$ et $1\mathcal{R}1$, thus $\mathcal{R}$ is reflexive.
2. For the reason that $0\mathcal{R}1$ and $1\mathcal{R}0$, so $\mathcal{R}$ is symmetric.
3. If $a\mathcal{R}b$ et $b\mathcal{R}c$, then $a\mathcal{R}c$. Per example $0\mathcal{R}0$, et $0\mathcal{R}\overline{1} \Rightarrow 0\mathcal{R}0$. We have only two elements in $\Omega_{\mathcal{R}}$, thus the relation is obvious.

We deduce that $\mathcal{R}$ is an equivalence relation : $\mathcal{R}_{\equiv}$.

Conjugate of imply symbol

Conjugate of equivalence symbol

We notice that :

$$\overline{a \, \mathcal{R}_{\equiv} \, b} = \overline{a} \, \overline{\mathcal{R}_{\equiv}} \, \overline{b} \tag{2}$$

Hence $(\overline{0} \; \overline{R}_{\equiv} \; \overline{0}) = (1 \; \overline{R}_{\equiv} \; 1)$, such that $1 \; \mathcal{R} \; 1$. Then, it's involving that $\mathcal{R}_{\equiv} = \overline{\mathcal{R}_{\equiv}}$, which is equivalent to write : $\overline{\Leftrightarrow} = \Leftrightarrow$.

Conjugate of imply symbol

Let $\Omega' = \{\Leftarrow, \Rightarrow\}$, then :

$$
\begin{aligned}
\Leftrightarrow &= (\Leftarrow \wedge \Rightarrow) \\
\text{is implying } \Lleftarrow\!\!\Rrightarrow &= \overline{\Leftarrow \wedge \Rightarrow} \\
&= \Lleftarrow \overline{\wedge} \Rrightarrow \\
&= \Lleftarrow \vee \Rrightarrow
\end{aligned} \tag{3}
$$

$$\text{As } \Leftrightarrow = \Lleftarrow\!\!\Rrightarrow, \text{ then : } \Leftrightarrow = \Lleftarrow \vee \Rrightarrow.$$

Thus :

$$
\begin{aligned}
\Leftrightarrow &= (\Leftarrow \wedge \Rightarrow) \\
&= (\Lleftarrow \overline{\wedge} \Rrightarrow)
\end{aligned} \tag{4}
$$

$$\text{cause } (\Leftarrow \wedge \Rightarrow) = (\Lleftarrow \vee \Rrightarrow)$$

Consequently : for a none exclusive either, let $\Leftarrow = \Lleftarrow \wedge \Rightarrow = \Rrightarrow$;
or $\Leftarrow = \Rrightarrow \wedge \Rightarrow = \Lleftarrow$.

Logic Connectors Contraposition Set Logic Coherence between formules Cretan Paradox

Conjugate of imply symbol

Order relation

Soient $\mathcal{O}_{\Rightarrow}$ et $\mathcal{O}_{\Leftarrow}$, two relations on $\Omega' = \{-1, 0, 1\}$. With $\overline{0} = 0$, $\overline{1} = -1$.

1. $0\mathcal{O}_{\Rightarrow}0$,$-1\mathcal{O}_{\Rightarrow} - 1$ et $1\mathcal{O}_{\Rightarrow}1$. Thus $\mathcal{O}_{\Rightarrow}$ is reflexive.
2. $0\mathcal{O}_{\Rightarrow}1$ et $-1\mathcal{O}_{\Rightarrow} - 0$ is involving $-1\mathcal{O}_{\Rightarrow}0$. Hence, the relation is anti-symmetric.
3. On another hand, $-1\mathcal{O}_{\Rightarrow}0$ and $0\mathcal{O}_{\Rightarrow}1$ which is involving $-1\mathcal{O}_{\Rightarrow}1$.So the relation $\mathcal{O}_{\Rightarrow}$ is transitive.

We deduce $\mathcal{O}_{\Rightarrow}$ is an order relation (generic, so un-strict).By an analogous reasoning, we have been to believe that $\mathcal{O}_{\Leftarrow}$is too an order relation.

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Conjugate of imply symbol

In term of symbol

We have $\mathcal{O}_{\Rightarrow}$ et $\mathcal{O}_{\Leftarrow}$ which are conjugates, in term of symbol :

$$\begin{cases} \mathcal{O}_{\Rightarrow} & = \overline{\mathcal{O}_{\Leftarrow}} = \mathcal{O}_{\Lleftarrow} \\ \mathcal{O}_{\Leftarrow} & = \overline{\mathcal{O}_{\Rightarrow}} = \mathcal{O}_{\Rrightarrow}. \end{cases}$$

Hence, in terms of connectors symbols :

$$\begin{cases} \Rightarrow & = \Lleftarrow \\ \Leftarrow & = \Rrightarrow. \end{cases}$$

Conjugate of imply symbol

$\Lleftarrow \Longrightarrow$ et $\Rrightarrow \Longleftarrow$

Contraposition

$A \Rightarrow B$, implies $\overline{A \Rightarrow B} = \overline{A} \Leftarrow \overline{B}$.

Theorem

$$A \Rightarrow B \Leftrightarrow \overline{B} \Rightarrow \overline{A}.$$

Corollaire

If $A \Rightarrow B = V$, *then* $\overline{A \Rightarrow B} = F \Leftrightarrow \overline{A} \Leftarrow \overline{B} = F$.

Corollaire

If $A \Rightarrow B = F$, *then* $\overline{A \Rightarrow B} = V \Leftrightarrow \overline{A} \Leftarrow \overline{B} = V$.

Set Logic

Let

$$\begin{aligned} \overline{A \setminus B} &= B \\ \overline{A \setminus B} &= \overline{A} \mathbin{\overline{\setminus}} \overline{B} \end{aligned} \tag{5}$$

If we have $\overline{\setminus} = \setminus$, then $\overline{A \setminus B} = \overline{A} \setminus \overline{B} = 0$.

As $B \neq 0$. We conclude :

$$\boxed{\setminus \neq \overline{\setminus}.}$$

A formule

Definition (Chaîne formelle)

A formal chain is a linguistinc form. It's an expression of a formal language.

Definition (Formule : Fr)

A formule Fr is a broadly expression of a relationship, between a formal chain and arguments which will return values.

Let $Fr(x_1, \ldots, x_n) = (y_1, \ldots, y_m)$. Fr a boolean formule, n_∞ ou 2^{n_∞} arguments. $Fr^{|}$ is a restriction, et par $Fr^{\star}$, is a prolongement.

De l'infini potentiel vers l'infini actuel

$$\begin{cases} \textit{If } Fr(x \mapsto 2^{n_\infty}) = F, \textit{then } Fr^\star(x = 2^{n_\infty}) = F \\ \textit{If } Fr(x \mapsto n_\infty) = F, \textit{then } Fr^\star(x = n_\infty) = F \end{cases}$$

Theorem

If a formule is wrong in the context of a potential infinite, it's wrong in the context of an actual infinite.

From actual infinite towards potential infinite

$$\begin{cases} \textit{If } Fr(x = 2^{n_\infty}) = V, \textit{then } Fr^{|}(x \mapsto 2^{n_\infty}) = V \\ \textit{If } Fr(x = n_\infty) = V, \textit{then } Fr^{|}(x \mapsto n_\infty) = V \end{cases}$$

Let $Fr^{|}(x \mapsto 2^{n_\infty}) = F \Rightarrow Fr^{|\star}(x = 2^{n_\infty}) = F$.

Theorem

If a formule is true in the context of an actual infinite, it's true in the context of a potential infinite.

Formulation of the Cretan Paradox

The cretan philosopher Epimenides is at the origin of the cretan paradox, wich has been revealed to the VI° century before J-C. Here is how it was submited.

1. All cretans are liars.
2. Epimenides the cretan tells that he is a liar.

We arrive to the conclusion : Epimenides lies and does not lie. We will define the referents joined to the statements. The referent is a fonction wich returns syntaxic items in natural language. Hence, for the statements system, we will have three referents.

1. r_1=Cretan.
2. r_2=Epimenide.
3. r_3=Liar.

The last one is for the conclusion.

Those statements will be put in a matching relation by $\mathcal{R}$. We define by $\overline{r_3}$ the conjugate referent (opposition) as : not a liar. We also indentify three states joined to the propositions and to the inferences.

1. $ST_1 = \forall r_1, r_1 \mathcal{R} r_3$.
2. $ST_2 = \exists r_2, r_2 \mathcal{R} r_1$.
3. $ST_3 = \exists r_2, r_2 \mathcal{R} r_3 \wedge r_2 \mathcal{R} \overline{r_3}$.

Undecidabilty

Definition (Undecidability)

A theory is called undecidable, if we can't show that we have either the statement p, or the statement $\bar{p}$.

We identify three kinds of undecidability :

1. The syntaxic undecidability (Ladrière).
2. The power undecidability (the NNT in an experimental study).
3. The entropic undecidability (which contains disorder and heterogeneity).

Undecidabilty

As a deduction on ST_3, we have Epimenide who is a liar, and Epimenide who is not a liar. We have $p \wedge \bar{p}$ wich lead to have an entropic undecidability of second specie.

Definition

If we show a statement is p and $\bar{p}$, the law of contradiction leads to a second specie of entropic undecidability.