Quantum Mechanics and Planck's Constant

Stratal foundations from the grain program

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Outline

The grain program so far

Three mathematical volumes:

- Volume I — *Axiomatics of the grain*: construction of $\widetilde{\mathbb{R}}$ and $\mathbb{R}$ from a geometric process (circles, angle doubling, stratification).
- Volume II — *Resolution symmetry*: bidirectional process, line at infinity Δ_∞, canonical bijection δ_k, double mosaic structure.
- Volume III — *Divergent integrals*: stratified integration, universal regularization at critical bound, hierarchy of divergences.

This talk: connecting to quantum mechanics

Two structural claims

Claim 1 (Quantum mechanics). The complex nature of the wave function, the Born rule, and the irreversibility of measurement are *geometric consequences* of the stratal duality.

Claim 2 (Planck constant). The constants h and $\hbar$ admit a *stratal interpretation*: h as energy of one intra-grain cycle, $\hbar$ as energy per radian. The factor 2π between them is the circumference of C_1.

Epistemological status. Exploratory parallel between suggestion and reformulation. We do not claim to derive quantum mechanics or the numerical value of h. We propose a stratal reading that grounds the mathematical structures geometrically.

The complex exponential emerges geometrically

Setup.

- Stratal logarithm $T = \log_2 : \widetilde{\mathbb{R}}_*^+ \to \widetilde{\mathbb{R}}$, sends $\mathcal{G}_{-2n_\infty}$ to $\mathcal{G}_{-n_\infty}$.
- Inverse: stratal exponential $E(\theta) = 2^\theta$.

Compatibility constraints with C_1.

- $\mathcal{E}(0) = +1$
- $\mathcal{E}(\pi) = -1$ (geometric antipode)
- $\mathcal{E}(2\pi) = +1$ (periodicity)
- $\mathcal{E}(\theta_1 + \theta_2) = \mathcal{E}(\theta_1) \cdot \mathcal{E}(\theta_2)$ (multiplicativity)

Real exponential fails: $E(\pi) = 2^\pi \neq -1$, not periodic.

Solution. Introduce factor ξ: $\mathcal{E}(\theta) = 2^{\xi\theta}$.

The forced emergence of i

Theorem (Geometric emergence of i)

The unique element ξ such that $\mathcal{E}(\theta) = 2^{\xi\theta}$ satisfies the circular constraints requires:

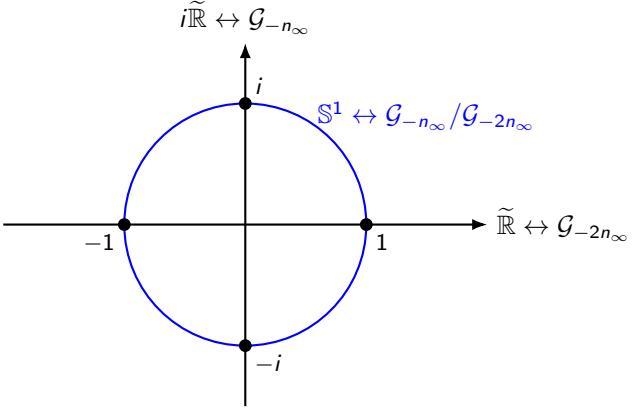
$$\boxed{\xi^2 = -1.}$$

This element is not in $\widetilde{\mathbb{R}}$: it belongs necessarily to an extension where $\sqrt{-1}$ exists. We call it i.

Proof sketch. By multiplicativity, $\mathcal{E}(\pi) = \mathcal{E}(\pi/2)^2$. With $\mathcal{E}(\pi) = -1$, we get $\mathcal{E}(\pi/2)^2 = -1$.

Key point. i is *not* algebraically adjoined. It is *geometrically forced* by the compatibility between the stratal exponential and the circular structure of C_1.

The stratal complex plane: three distinct objects



Stratal superposition principle. The unit circle $\mathbb{S}^1$ is not geometrically distinct from C_1: it is the same support, with the quotient stratal structure. Three objects on the same physical support, with distinct stratal roles.

In the laboratory

Experimental Set

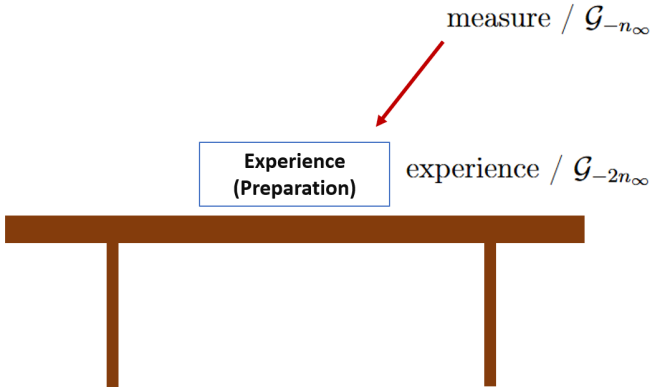


Figure: Relation Math-Physics

The wave function as stratal object

Definition (Stratal wave function)

A *stratal wave function* is a map $\psi : \widetilde{\mathbb{R}}_*^+ \to \mathscr{P}_{\mathbb{C}}$, with $\psi(x) = |\psi(x)| \cdot e^{i\theta(x)}$ where:

- Amplitude $|\psi(x)| \in \widetilde{\mathbb{R}}_*^+$: *fine stratum* $\mathcal{G}_{-2n_\infty}$ (real axis).
- Phase $e^{i\theta(x)} \in \mathbb{S}^1$: *cyclic quotient* $\mathcal{G}_{-n_\infty}/\mathcal{G}_{-2n_\infty}$ (unit circle).

Reading.

- Amplitude lives where positions are precisely localized (fine stratum).
- Phase lives on the unit circle, which is intrinsically cyclic (the quotient stratal structure).

Probabilistic character: stratal cardinality

Stratal probability principle

The probabilistic nature of quantum mechanics emerges from *stratal cardinality*: 2^{n_∞} fine positions project onto each coarse position. The observer at coarse resolution cannot access individual fine positions.
The probability of a measurement outcome is proportional to the number of fine positions compatible with that outcome.

Information loss as fundamental.

- Fine positions remain hidden to the coarse observer.
- Their multiplicity produces the probabilistic character.

The Born rule: emergence of $|\psi|^2$

Theorem (Stratal Born rule)

If amplitude $|\psi|$ lives in the fine stratum $\mathcal{G}_{-2n_\infty}$, the *probability density* observed at coarse resolution is $|\psi|^2$ as a consequence of the factor 2 between adjacent strata.

Why the square? By lexical multiplication (Volume I), for $x \in \mathcal{G}_{-n_\infty}$, the product $x \cdot x = x^2$ lives in $\mathcal{G}_{-2n_\infty}$. The square is the *strata transition* between observer (coarse) and reality (fine).

Conceptual reading. The exponent 2 is not arbitrary:

- Classical (same stratum): probability $\propto |\psi|$.
- Quantum (one stratum shift): probability $\propto |\psi|^2$.
- Hypothetical (two stratum shifts): probability $\propto |\psi|^4$.

The Born rule reflects the *depth of stratification* characteristic of quantum mechanics.

Measurement as stratal projection

Stratal measurement principle

Quantum measurement is the stratal projection

$$\pi : \mathcal{G}_{-2n_\infty} \longrightarrow \mathcal{G}_{-n_\infty}.$$

This projection is *irreversible*: 2^{n_∞} fine positions compatible with each coarse outcome remain indistinguishable.
Before measurement. Superposition = plurality of fine positions encoded by amplitude and phase.
After measurement. Coarse position determined; phase is erased (the unit circle is collapsed to a point).

Wavefunction collapse as scale transition. No physical collapse: a *change of observation scale* aggregating fine plurality into a unique coarse outcome.

Space and time emerge from the process

Stratal spatial-temporal principle

- **Space** is measured by inter-grain displacements: moving between grains of $\mathcal{G}_{-2n_\infty}$ (real axis).
- **Time** is measured by intra-grain displacements: rotation in the quotient $\mathcal{G}_{-n_\infty}/\mathcal{G}_{-2n_\infty}$ (unit circle).

Physical units grounded stratally.

- Meter $\leftrightarrow$ reference inter-grain displacement.
- Second $\leftrightarrow$ reference intra-grain cycle.

Velocity.

$$v = \frac{\Delta x_{\text{inter}}}{\Delta t_{\text{intra}}}.$$

The speed of light as stratal velocity

Stratal speed of light

The speed of light c is the maximal spatial velocity: a grain at most traverses one grain-length during one intra-grain cycle.

$$c = \frac{\text{grain length}}{\text{intra-grain cycle duration}}.$$

Why c is constant. Both grain length and intra-grain cycle duration are *structural* stratal quantities — they are not parameters of the program, they are constitutive.

Universal and mirror flows. The flows Γ and $\bar{\Gamma}$ trace the carrier circles C_k at speed c, which become naturally *the circles of light propagation* (Feynman's QED imagery).

h as energy of one intra-grain cycle

Theorem (h as stratal energy)

The Planck constant h identifies with the fundamental energy of one complete intra-grain cycle at level $\mathcal{G}_{-n_\infty}$:

$$\boxed{h = \mathcal{E}_{-n_\infty}.}$$

The Planck-Einstein relation $E = h\nu$ expresses directly that frequency counts cycles per unit time, and h is the energy per cycle.

Geometric meaning.

- One full revolution on C_1 (= one intra-grain cycle) costs energy h.
- Multiple revolutions per unit time: ν cycles per second.
- Total energy: $E = h \cdot \nu$ — exactly Planck-Einstein.

$\hbar$ as energy per radian

Theorem ($\hbar$ from circumference of C_1)

The reduced Planck constant is the energy per radian:

$$\boxed{\hbar = \frac{h}{2\pi} = \frac{\mathcal{E}_{-n_\infty}}{\text{circumference of } C_1}.}$$

The factor 2π is not a convention.

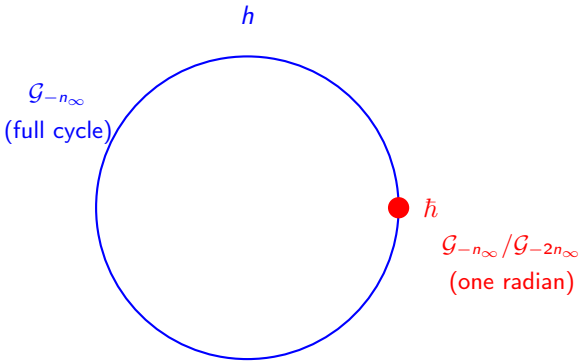
- Classical view: $\hbar = h/2\pi$ is a notational choice for angular frequencies.
- Stratal view: 2π is the *circumference of* C_1, the reference circle of the program.

***h* vs $\hbar$.**

- h: energy per complete cycle (natural for ν in Hz).
- $\hbar$: energy per radian (natural for ω in rad/s).

The ratio $h/\hbar = 2\pi$ is a *geometric invariant* of the program.

Stratal levels of h and $\hbar$



Stratal superposition. Both h and $\hbar$ live on the same geometric support (circle C_1) but with distinct stratal structures:

- h: quantity associated to the whole cycle.
- $\hbar$: quantity associated to one radian (one « point » at coarse view, a small structure at fine view).

Numerical identification in SI units

What the program gives us:

1. h as energy of one intra-grain cycle (structural definition).
2. $h/\hbar = 2\pi$ as geometric consequence (circumference of C_1).
3. Stratal grounding of space, time, and velocity units.

What it does *not* give:

- Absolute derivation of $h \approx 6.626 \times 10^{-34}$ J·s.

Operational identification. The numerical value in SI units is a *conversion coefficient* between stratal quantities and conventional units (joule, second). Since 2019, h is fixed by convention in SI: $h = 6.62607015 \times 10^{-34}$ J·s exactly.

The stratal program identifies the *structure*; the numerical identification is a separate matter of unit conventions.

Conjecture: matter and antimatter

Conjecture (Matter/antimatter as dual flows)

The universal flow Γ (clockwise) corresponds to matter propagation. The mirror flow $\bar{\Gamma}$ (counter-clockwise) corresponds to antimatter propagation. The two flows are mirror images by axial symmetry across Δ, combined with orientation inversion.
Annihilation corresponds to the meeting of both flows on Δ, producing phase decoherence.

Limits of the conjecture.

- Physical matter/antimatter difference: charge, quantum numbers, CPT behavior.
- Pure « flow orientation inversion » does not capture these quantitative aspects.
- Open directions: stratal definition of charge, articulation with discrete symmetries, derivation of pair creation/annihilation.

Other conjectures and open directions

Unitary evolution as stratal rotation.

$$e^{-i\hat{H}t/\hbar} \quad = \text{rotation on } \mathbb{S}^1.$$

$\hat{H}$ generates rotation, $\hbar$ measures stratal time scale.

Heisenberg uncertainty as stratal invariant. $\Delta X \cdot \Delta P \geq \hbar/2$ may relate to the $(2\pi)^2$ invariant of Volume II (crossed product of fine and coarse finenesses).

Other directions.

- Articulation with relativity (stratal spacetime).
- Identification of other constants (G, α).
- Multi-particle systems and quantum field theory.
- Formalization of mirror flow dynamics.

What the stratal reading accomplishes

1. **Complex nature of ψ:** not an algebraic axiom, but a geometric consequence of stratal duality. i emerges as forced by circular compatibility.
2. **Born rule $|\psi|^2$:** consequence of the factor 2 between adjacent strata. The square reflects the depth of stratification between observer and reality.
3. **Measurement irreversibility:** stratal projection $\mathcal{G}_{-2n_\infty} \to \mathcal{G}_{-n_\infty}$, with information loss by aggregation of fine positions.
4. **Space, time, velocity:** grounded in inter-grain / intra-grain displacements. The speed of light is the maximal stratal velocity.
5. **Planck constants:** h = energy of one intra-grain cycle, $\hbar$ = energy per radian. The factor 2π is the circumference of C_1 — a geometric invariant.

What remains conjectural

- **Numerical value of h:** the stratal identification is operational, not derivational. SI value remains conventional.
- **Matter/antimatter from flow duality:** conceptually suggestive but needs articulation with charge, quantum numbers, and CPT symmetries.
- **Schrödinger equation:** the connection between Hamiltonian and stratal rotation generator needs precise formalization.
- **Articulation with relativity:** the stratal spacetime framework remains to be developed.
- **Multi-particle systems:** entanglement, tensor products, and quantum field theory need stratal interpretation.

These directions open an autonomous research program in the prolongation of the grain program.

Thank you.

Questions?
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