

Instants and Moments

Time in Grain Theory

Hugues Genvrin

September 2026

Overview

One question: *is time made of instants?*

1. Instants and moments

- at each resolution, time is made of **moments**: grains of infinitesimal duration
- an **instant** is the promotion of the moments that contain it

2. Zeno's arrow — the arrow moves **during every moment**

3. Special relativity

- proper time is a **stratification** of coordinate time, of factor $1/\gamma$
- Langevin's twins are a **homogeneous count** of moments

4. Lorentz invariance — broken by grids, but only **infinitesimally**; exact for instants

Time is made of **moments**; instants exist only after **promotion**.

Outline

Grain theory: reminders

I. Instants and moments

II. Zeno's arrow

III. Proper time as a stratification

IV. Langevin's twins

V. Lorentz invariance

Synthesis

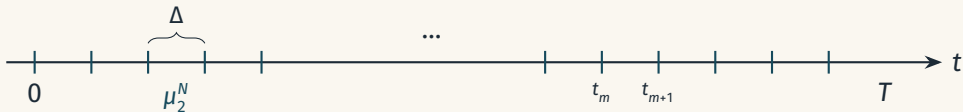
Grain theory: reminders

Time grids, junctions and promotion

Let n_∞ be a hyperinteger, $k \geq 1$, $N = kn_\infty$, and T a standard duration. Set

$$\Delta = T 2^{-N} \text{ (infinitesimal),} \quad \mu_m^N = [m\Delta, (m+1)\Delta[, \quad 0 \leq m < 2^N.$$

- The grains μ_m^N cut $[0, T]$; their endpoints $t_m = m\Delta$ are the **junctions**.
- **Promotion** $\tilde{\mathbb{R}}_{\text{fin}}/\mathfrak{m} = \mathbb{R}$ is realized by the standard part **st**. We write $a \approx b$ (*infinitely close*) when $a - b$ is infinitesimal.
- **Transfer**: first-order statements true for all integers remain true for hyperintegers.



Two earlier results we shall use

Theorem 1 (Functional stratification)

A differentiable f maps $[x, x + dx]$ onto $[f(x), f(x) + f'(x) dx]$: it acts on its target axis as a **local stratification factor** $|f'(x)|$.

Theorem 2 (Nested adjacent dyadic segments)

Every real number is the limit of the nested segments whose endpoints are **adjacent dyadic numbers**; these segments are its **grains**, at every resolution.

In this talk, f will be a **position** $x(t)$ or a **proper time** $\tau(t)$,
and the real number will be an **instant**.

I. Instants and moments

An old question: is time made of instants?

- **Classical physics.** Time is the real line: an instant is a point, and a duration is an uncountable set of instants.
- **Aristotle** (*Physics*, IV and VI). The “now” is not a part of time but a **limit**, as the point is not a part of the line. Time is made of durations.
- **Whitehead** (1934). There is **no nature at an instant**: every physical process requires a duration.
- **Zeno’s arrow** exploits precisely a time composed of instants without duration.

What grain theory offers

A precise form for this intuition:

- at a given resolution, time grains are **moments**;
- at the next resolution, each moment splits into two finer moments;
- true **instants** exist only after promotion.

Moments and their division

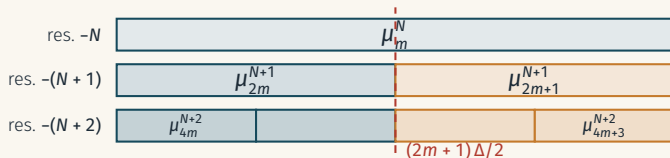
Definition 3 (Moment)

A **moment** of resolution $-N$ is a time grain μ_m^N . Its *duration* is $\Delta = T 2^{-N}$. The junctions t_m are the **boundaries** between consecutive moments.

Proposition 4 (Division of moments)

At resolution $-(N + 1)$, the moment μ_m^N splits into exactly two moments, μ_{2m}^{N+1} and μ_{2m+1}^{N+1} , separated by the boundary $(2m + 1) \Delta/2$.

Proof: $\mu_m^N = [2m \frac{\Delta}{2}, (2m + 2) \frac{\Delta}{2}] = \mu_{2m}^{N+1} \cup \mu_{2m+1}^{N+1}$.



The instant as the standard part of a moment

Proposition 5 (Instant)

If N is unlimited, the two boundaries of a moment μ_m^N are infinitely close. Hence we may set

$$\text{st}(\mu_m^N) := \text{st}(m\Delta) = \text{st}((m+1)\Delta),$$

which gives a **surjective** map from moments onto the standard instants of $[0, T]$. Two moments have the same instant if and only if their boundaries are infinitely close.

Proof: both boundaries lie in $[0, T]$, hence are limited, and they differ by $\Delta \approx 0$. For a standard $t \in [0, T]$, the moment containing t has boundaries within Δ of t ; for $t = T$ take the last moment.

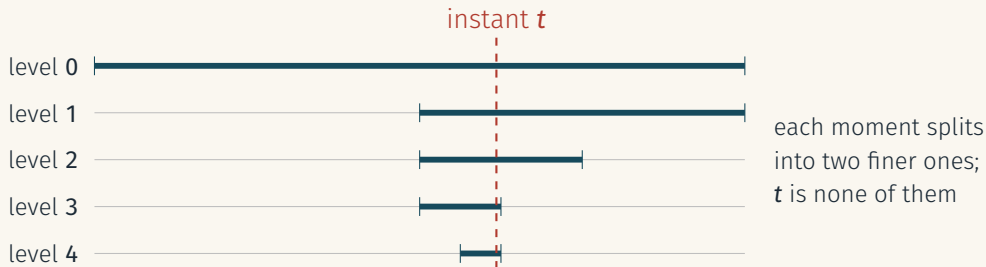
Definition 6 (Instant)

An **instant** is a standard element $t \in [0, T]$, that is, the common standard part of a family of infinitely close moments.

The tower of moments around an instant

Corollary 7 (Instants are promotions of nested moments)

For every instant $t \in [0, T[$, the moments containing t at resolutions $-N, -(N+1), -(N+2), \dots$ are **nested**, their boundaries are **adjacent junctions**, and each has standard part t .



Aristotle made precise

An instant is not a moment

- every moment has a non-zero duration Δ ;
- an instant, being promoted, has duration $\text{st}(\Delta) = 0$;
- a standard duration $[a, b]$ contains about $(b - a)/\Delta$ moments, an **unlimited hyperinteger**.

Boundaries are not instants

The junctions t_m look like instants, but they are hyperrationals, in general non-standard, separating two moments. Only promoted instants are instants in the sense of classical physics.

Space and time, side by side

Line (cuts)	Time
grain	moment
junction	boundary
point	instant
point = promotion of nested grains	instant = promotion of nested moments
line not made of points	time not made of instants

II. Zeno's arrow

The arrow paradox

1. At each instant, the arrow occupies a space equal to itself: it is **at rest**.
2. The time of flight is **composed of instants**.
3. Therefore the arrow is always at rest.



Russell's answer (1903): the **at-at theory**.

- Motion is nothing but being at different places at different instants.
- It makes no sense to speak of motion *at* an instant.
- Instantaneous velocity is a **limit**.

The arrow moves during every moment

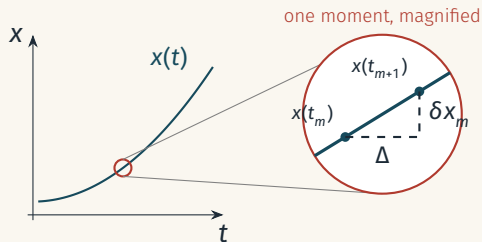
Let $x(t)$ be the standard position of the arrow, of class C^1 , and $v = x'$.

Proposition 8 (The arrow moves)

Let N be unlimited and μ_m^N a moment whose instant $t = \text{st}(\mu_m^N)$ satisfies $v(t) \neq 0$. Then the displacement during this moment, $\delta x_m = x(t_{m+1}) - x(t_m)$, is **non-zero**, and

$$\text{st}\left(\frac{\delta x_m}{\Delta}\right) = v(t).$$

Proof: by the (transferred) mean value theorem, $\delta x_m = v(\xi) \Delta$ with ξ between t_m and t_{m+1} . Since $\xi \approx t$ and v is continuous, $v(\xi) \approx v(t) \neq 0$.



Zeno's premise falls

In the reading by moments

The time of flight is not composed of instants but of **moments**, and **during every moment, the arrow moves**.

- Motion exists at every resolution; only the position *at* an instant requires promotion.
- Instantaneous velocity is not the limit of an endless process but the **standard part of a ratio computed on a single moment**, as in the study of free fall.

	Russell (at-at)	Moments
Level	promoted time	grids
Motion	instants $\mapsto$ positions	$\delta x_m \neq 0$ in each moment
Velocity	a limit	$\text{st}(\delta x_m / \Delta)$

The two answers **do not conflict**: Russell's describes promoted time, the moments describe the grids.

III. Proper time as a stratification

Proper time and the Lorentz factor

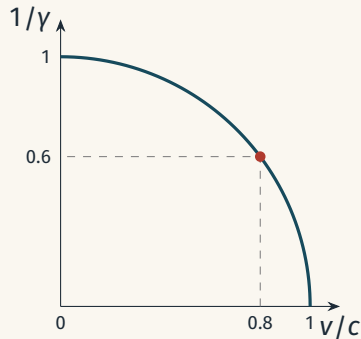
A clock moving at velocity $\mathbf{v}(t)$, $|\mathbf{v}| < c$, in an inertial frame S measures the **proper time**

$$\tau(t) = \int_0^t \frac{ds}{\gamma(s)}, \quad \gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \geq 1.$$

Remark 9 (What dilation is not)

Time dilation relates the proper times of clocks in relative motion. It does **not** depend on the resolution: cutting time into moments neither produces nor explains it. Conversely, it does not prove that time is granular.

The link with grains lies elsewhere: in how the moments of one observer are read in the moments of another.



$1/\gamma = \sqrt{1 - v^2/c^2}$ is a **quarter circle**.

At $v = 0.8c$: $1/\gamma = 0.6$, $\gamma = 5/3$.

Proper time is a stratification of coordinate time

Proposition 10 (Stratification by $1/\gamma$)

Cut the coordinate time of S into moments of duration Δ . During μ_m^N , the moving clock measures the proper duration

$$\delta\tau_m = \tau(t_{m+1}) - \tau(t_m), \quad \text{st}\left(\frac{\delta\tau_m}{\Delta}\right) = \frac{1}{\gamma(t)}, \quad t = \text{st}(\mu_m^N).$$

If the velocity is constant, $\delta\tau_m = \Delta/\gamma$ exactly.

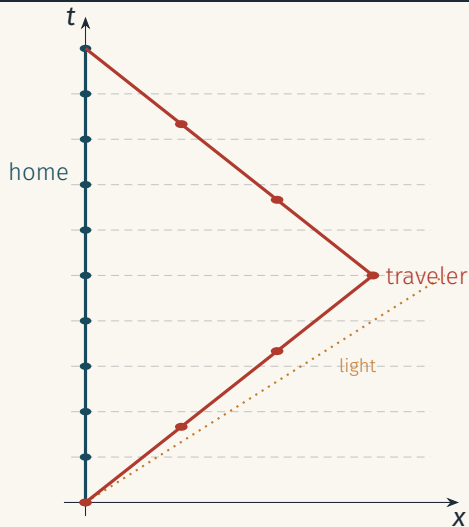
Proof: the arrow proposition applied to τ , whose derivative $1/\gamma$ is continuous and non-zero.



Functional stratification with factor $d\tau/dt = 1/\gamma$. In free fall, velocity *thickened* space grains; here motion *thins* proper moments. Conversely, one proper moment Δ covers $\approx \gamma\Delta$ of coordinate time: *what dilates are the moments*.

IV. Langevin's twins

Langevin's twins in a spacetime diagram



- The twins part at instant 0 and meet again at instant T of the frame S .
- The home twin stays at rest in S : **10** units of proper time (intervals between dots).
- The **traveler** goes out and back at $v = 0.8c$: only **6** units, one every $\gamma = 5/3$ units of coordinate time.
- Both cross the **same coordinate moments** (dashed lines).

Units: $c = 1$, $T = 10$. Out to $x = 4$ at $t = 5$, back at $t = 10$.

Same number of moments, different thicknesses

The traveler follows a trajectory of continuous velocity $\mathbf{v}(t)$, not identically zero.

Proposition 11 (Homogeneous count)

During the journey, both twins go through the **same 2^N moments** of coordinate time. But if their proper durations are counted in moments of the same duration Δ (**homogeneous count**),

$$N_{\text{home}} = 2^N, \quad N_{\text{trav}} = \left\lfloor \frac{\tau(T)}{\Delta} \right\rfloor, \quad \text{st}\left(\frac{N_{\text{trav}}}{N_{\text{home}}}\right) = \frac{1}{T} \int_0^T \frac{dt}{\gamma(t)} < 1.$$

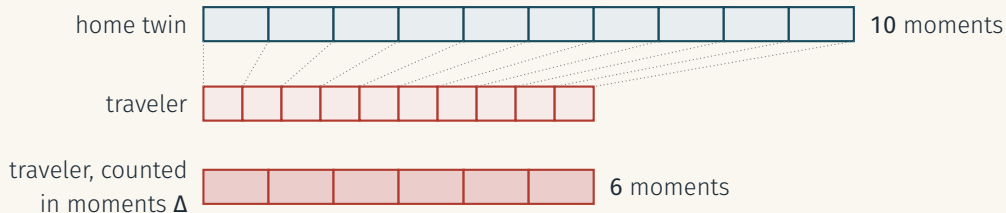
The gap $N_{\text{home}} - N_{\text{trav}}$ is an **unlimited** hyperinteger.

Proof:

- the home twin's proper time is $T = 2^N \Delta$;
- the traveler's is $\tau(T) = \int_0^T dt/\gamma < T$, since $1/\gamma \leq 1$, strictly where $\mathbf{v} \neq 0$;
- $N_{\text{home}} - N_{\text{trav}} \geq (T - \tau(T))/\Delta$: a standard positive number divided by an infinitesimal.

Example: a round trip at $v = 0.8c$

Neglecting the acceleration phases, $\tau(T) = T/\gamma$. For $v = 0.8c$, $\gamma = 5/3$: the traveler counts **3/5** of the home twin's moments.



With **10** moments for the picture: both twins cross the same moments of coordinate time, but the traveler's proper moments are thinner, of thickness Δ/γ . Counted in moments of duration Δ , the traveler has lived only **6** of them.

The twins as a homogeneous count, as in reflexivity

	Reflexivity	Twins
Two objects	part $\mathcal{F}_2^*$, whole $\mathcal{F}_1$	proper times of traveler, home twin
Grain-to-grain map	$y \mapsto ay$	same coordinate moments
Stratification factor	Jacobian a	$1/\gamma$
Same number of grains	2^{n_∞} and 2^{n_∞}	2^N and 2^N
Homogeneous count	$2^{n_\infty} < \lfloor a 2^{n_\infty} \rfloor$	$N_{\text{trav}} < N_{\text{home}}$
Standard ratio	$1/a$	$\frac{1}{T} \int_0^T dt/\gamma$

Same number of grains, different thicknesses: it is the **homogeneous count** that reveals the difference.

What grains do not claim

Grains give no new *explanation* of the twin paradox. Its asymmetry comes, as in ordinary relativity, from the traveler's **change of inertial frame**. Grains give a **counting reading** of it.

V. Lorentz invariance

A spacetime grid privileges a frame

Take $c = 1$, dimension $1 + 1$. In the frame S , cut spacetime into square grains: the junctions are the events

$$P_{m,n} = (m\Delta, n\Delta), \quad m, n \text{ hyperintegers.}$$

A Lorentz transformation of velocity β , $|\beta| < 1$:

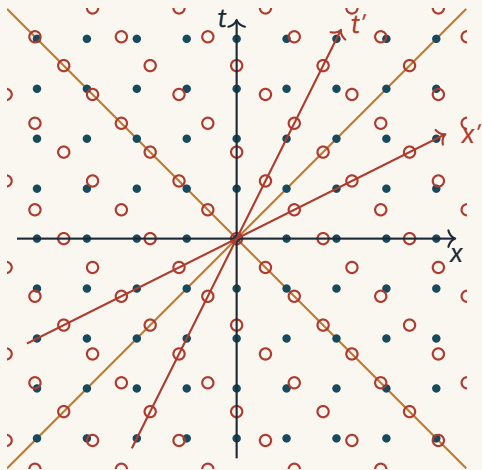
$$\Lambda_\beta(t, x) = (\gamma(t - \beta x), \gamma(x - \beta t)), \quad \gamma = (1 - \beta^2)^{-1/2}.$$

Proposition 12 (Non-aligned grids)

Let β be standard and non-zero.

1. The image under Λ_β of the grid of S is a **diamond lattice**, not the square grid of step Δ of S' .
2. If β is irrational, the world line of the origin of S' meets **no junction** of S other than the origin.

The two grids, seen from S



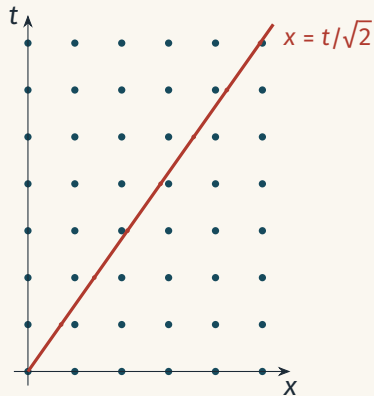
- Filled dots: junctions of S (square grid).
- Circles: junctions of S' , seen in S , for $\beta = 0.5$.
- The grid of S' is a diamond lattice, with axes tilted towards the light cone.
- Images of the basis vectors: $\gamma\Delta(1, -\beta)$ and $\gamma\Delta(-\beta, 1)$, not orthogonal in the Euclidean sense.

A grid privileges a frame.

Irrational velocity: no shared junction

Proof of point 2. The world line of the origin of S' is $x = \beta t$, and $P_{m,n}$ lies on it iff $n = \beta m$. For standard irrational β , “for all integers $m \neq 0$ and n , $n \neq \beta m$ ” is true; by **transfer** it holds for hyperintegers.

- A clock at rest in S' **never meets** a junction of S .
- Same situation as the **incommensurable grids** of the reflexivity study, where an irrational factor prevented grains from nesting.
- The classical difficulty of discrete spacetimes; **causal sets** (Bombelli et al. 1987) avoid it with random sprinklings.



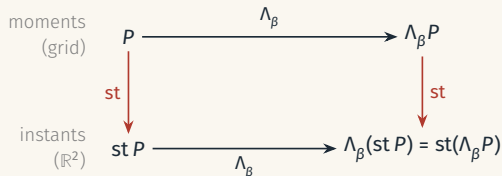
$\beta = 1/\sqrt{2}$: the world line crosses each row of junctions between two of them.

Lorentz invariance restored for instants

Theorem 13 (Lorentz invariance at the level of instants)

Let β be standard, $|\beta| < 1$.

1. For every junction P of S with limited coordinates, $\text{st}(\Lambda_\beta P) = \Lambda_\beta(\text{st } P)$.
2. The grids of S and S' , restricted to their limited junctions, have the **same promotion**: the whole plane $\mathbb{R}^2$.
3. For every standard event E , if P is the junction of S nearest to E , then $\Lambda_\beta P \approx \Lambda_\beta E$: the error due to the grid is **infinitesimal**.



Proof: (1) Λ_β is linear with standard coefficients; if $P = \text{st } P + \iota$, $\iota \approx 0$, then $\Lambda_\beta \iota \approx 0$. (2) The instant proposition, coordinate by coordinate. (3) $|P - E| \leq \sqrt{2} \Delta$, and Λ_β stretches Euclidean lengths by a bounded standard factor.

Broken for moments, exact for instants

- **At the level of moments**, frames are not equivalent: their grids differ, and may share no junction along a world line.
- **At the level of instants**, after promotion, every difference vanishes, and the Lorentz group acts **exactly** on $\mathbb{R}^2$.

The general pattern of our work: what depends on the grid — here, the **choice of a frame** — is **erased by promotion**.

Scope of the result

The theorem is **kinematic**: it concerns events and their coordinates. A physical law written on the grid (for instance a discretized wave equation) generally contains terms that violate Lorentz invariance. These terms are infinitesimal, but showing that they vanish at promotion for the **dynamics** requires a case-by-case study: a natural **open question**.

Synthesis

Two levels of time

	Moments (grids)	Instants (promoted)
Duration	$\Delta = T 2^{-N}$, infinitesimal, > 0	0
Time is made of	2^N moments, each splitting in two	not composed of instants
Zeno's arrow	moves during each moment	$v(t) = \text{st}(\delta x_m / \Delta)$
Moving clock	proper moments thinner by $1/\gamma$	$\tau = \int dt / \gamma$
Twins	same count, different thicknesses	$\tau(T) < T$
Frames	grids differ (diamond lattice)	Lorentz group acts exactly

In one sentence

Moments **dilate** from one frame to another, while instants, being promoted, **transform exactly** under the Lorentz group.

Results

1. An instant is the **common promotion** of the moments that contain it: a precise form of Aristotle's thesis.
2. Zeno's arrow: during each moment the arrow **moves**, and velocity is the promotion of a ratio computed on **one** moment.
3. Proper time is a **stratification** of coordinate time of factor $1/\gamma$: what dilates are the moments.
4. The twin paradox is a **homogeneous count** of moments, as in the reflexivity study.
5. A spacetime grid breaks Lorentz invariance, but only **infinitesimally**; at the kinematic level, invariance is exact for instants.

Relativistic dilation does not prove that time is granular, and grains do not explain dilation. But the two **agree**.

One pattern across the program

Study	Sharp on the grid	After promotion
Reflexivity	homogeneous count: part < whole	equal cardinals
Axiom of choice	canonical choice: first grain	only the infimum survives
Cuts	no gaps; position θ in the grain	Dedekind gaps appear
Free fall	odd-number law, exact	$x = \frac{1}{2}gt^2$
Time	moments; a preferred frame	instants; exact Lorentz invariance

Each time, the grid carries a **discrete, exact structure**, and promotion turns it into the classical continuous notion, **erasing** what depended on the grid.

Open questions

1. **Dynamics.** Do the Lorentz-violating terms of a law written on the grid (e.g. a discretized wave equation) always vanish at promotion?
2. **Regular or random grids?** Causal sets restore invariance with random sprinklings; grains keep regular grids and restore it by promotion. How do the two strategies compare?
3. **Gravitational time dilation.** In a static field, $d\tau/dt = \sqrt{1 - 2GM/(rc^2)}$ is another stratification factor. Does the homogeneous-count reading extend to general relativity?
4. **Physical scale.** The duration Δ is infinitesimal, not a Planck time. Is there any physical meaning in a finite resolution, or are moments purely a mathematical level?

References i

-  H. Genvrin, *Théorie du grain*, manuscript, 2026.
-  H. Genvrin, *Instants et moments : le temps dans la théorie du grain*, 2026.
-  H. Genvrin, *Le paradoxe de la réflexivité dans la théorie du grain*, 2026.
-  H. Genvrin, *Théorie des coupures et théorie du grain*, 2026.
-  H. Genvrin, *La chute libre : une interprétation physique de la théorie du grain*, 2026.
-  Aristotle, *Physics*, books IV and VI.
-  A. N. Whitehead, *Nature and Life*, Cambridge University Press, 1934.
-  B. Russell, *The Principles of Mathematics*, Cambridge University Press, 1903, chapter LIV.

-  A. Einstein, “Zur Elektrodynamik bewegter Körper”, *Annalen der Physik* 17 (1905), 891–921.
-  P. Langevin, “L'évolution de l'espace et du temps”, *Scientia* 10 (1911), 31–54.
-  L. Bombelli, J. Lee, D. Meyer, R. D. Sorkin, “Space-time as a causal set”, *Physical Review Letters* 59 (1987), 521–524.
-  A. Robinson, *Non-standard Analysis*, North-Holland, 1966.